

Temperature of the Central Stars of Planetary Nebulae and the Effect of the Nebular Optical Depth

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ABSTRACT

The effect of the nebula optical depth on the determination of the temperature (T_*) of the central stars in planetary nebulae is discussed. Based on photoionization models for planetary nebulae with different optical depths, we show, quantitatively, that the details of the distribution of the H and He II Zanstra temperatures are mainly explained by an optical depth effect; in particular, that the discrepancy is larger for low stellar temperatures. The results also show that for high stellar temperatures the He II Zanstra temperature underestimates the stellar temperature, even for high optical depths. The stellar temperature, as well as the optical depth, can be obtained from a Zanstra temperature ratio (ZR) plot $ZR = T_Z(\text{He II})/T_Z(\text{H})$ versus $T_Z(\text{He II})$. The effects of departures from a blackbody spectrum, as well as of the He abundance in the nebulae, are also discussed. For nebulae of very low optical depth and/or high stellar temperature the distribution ZR versus $T_Z(\text{He II})$ only provides lower limits for T_* . In order to obtain better values for the optical depth and T_* , we propose the use of the line intensity ratio He II/He I versus $T_Z(\text{He II})$ diagram.

Subject headings: planetary nebulae: general — stars: AGB and post-AGB-stars: fundamental parameters

1. Introduction

The temperature of the central stars of planetary nebulae (PNs) is an essential parameter for evolutionary studies as well as for an analysis of the nebulae themselves. The different methods for the determination of the stellar temperature and the problems involved were extensively discussed by Pottasch (1984) and Kaler (1985a, 1989). The most common method was suggested by Zanstra (1931) and further developed by Harman & Seaton (1966). The temperature of the ionizing star of a planetary nebula is calculated from the ratio between the flux of a recombination line and the stellar continuum flux at a given frequency. The Zanstra method assumes that all photons above the H (or the He^+) Lyman limit are absorbed within the nebula and that each recombination eventually results in a Balmer photon. The total ionizing flux can then be related to the total flux of a recombination line. The Zanstra method yields then two values for the stellar temperature: the Zanstra temperature obtained from the intensity of a hydrogen recombination line, $T_Z(\text{H})$, and the Zanstra temperature obtained from a He II recombination line, $T_Z(\text{He II})$. When applied to observed nebulae, $T_Z(\text{H})$ is generally lower than $T_Z(\text{He II})$; the difference can reach values of the order of 60,000 K (Kaler 1983b), and the temperature ratio can reach a factor higher than 3 (Kaler 1983b, 1985a). This is the well known Zanstra discrepancy, which is stronger for PNs with low stellar temperatures. In fact, many PNs with $T_Z(\text{H}) < 100,000$ K have $T_Z(\text{He II}) > T_Z(\text{H})$ while, for higher stellar temperatures, both Zanstra temperatures are similar (Pottasch, 1984; Gathier & Pottasch 1988, 1989). Another important point is that the calculated Zanstra temperatures do not reproduce the high temperatures predicted by evolutionary models (Kaler 1985a, 1989; Stasińska & Tylenda 1986).

Another interesting feature that could be related to the Zanstra temperature issues is the distribution of objects in a $\log L - \log T$ plane. As noted by Shaw & Kaler (1989), when this distribution is based on Zanstra temperatures there is a dense crowd of planetary nebula

nuclei with temperatures of $\sim 100,000$ K and a strong decrease towards higher temperatures, what these authors call the “Zanstra wall”.

The causes of the Zanstra discrepancy have been discussed by several authors (Pottasch 1984; Kaler 1985a, 1989; Henry & Shipman 1986; Stasińska & Tylenda 1986; Kudritzki & Méndez 1989; Gabler, Kudritzki & Méndez 1991; Méndez, Kudritzki & Herrero 1992) and can be related to the following effects: (1) optical effects, i.e., nebulae exhibiting a Zanstra discrepancy would be optically thin to photons ionizing H yet optically thick to those ionizing He^+ ; (2) differential dust absorption in the nebula; (3) the stellar continuum differing from the usually assumed blackbody spectrum; for example, an excess of photons with energies beyond the He^+ ionization potential would result in a high He II Zanstra temperature.

Objects with a large discrepancy show fainter low-ionization lines, suggesting that the effect 1 is the correct interpretation (Kaler 1983b). Simple evolutionary models of planetary nebulae predict $T_Z(\text{H}) \neq T_Z(\text{He II})$ during the nebula lifetime because of the variation of their optical depth (Tylenda et al. 1994; see also Schönberner & Tylenda 1990). The effect of dust on the calculated Zanstra temperature is discussed by Helfer et al. (1981). For an assumed absorption law (with a high opacity around 50 eV), Zanstra temperatures underestimate the actual stellar temperature. The effect is more intense for $T_Z(\text{H})$ and leads to the Zanstra discrepancy. However, dust appears to be only important in some specific nebulae (Kaler 1985a). Departures from the blackbody, in particular an excess of photons beyond the He^+ ionization potential, are suggested by observational studies of many central stars of planetary nebulae (Kaler 1985a; see also references in Henry & Shipman 1986). An ionizing spectrum with an excess of high energy photons (relative to a blackbody), produced by a star with a less than solar atmospheric He abundance, leads to a He II Zanstra temperature higher than $T_Z(\text{H})$ (Henry & Shipman 1986).

Detailed theoretical analyses, using photoionization models and applied to optically

thick nebulae, are presented by Stasińska & Tylenda (1986) and Henry & Shipman (1986). Stasińska & Tylenda (1986) show that for low stellar temperatures ($T_* \leq 100,000$ K), both Zanstra temperatures are similar. For higher stellar temperatures, $T_Z(\text{H})$ is larger and $T_Z(\text{He II})$ is lower than T_* . However, this result is opposite to what is obtained from observations. Analyzing only models with $T_* \leq 150,000$ K, Henry and Shipman (1986) conclude that $T_Z(\text{H})$ is a good measure of the stellar temperature. In fact, for these values of T_* , $T_Z(\text{H})$ and $T_Z(\text{He II})$ are similar (Stasińska & Tylenda 1986) and provide a good stellar temperature estimation *if the nebula is completely optically thick*.

Other methods for determining the stellar temperature include modelling of stellar absorption line profiles, ionic ratios, fitting of model atmospheres, the energy balance or Stoy method, stellar UV energy distribution, etc. The results from these different methods were compared with the values given by the Zanstra method in order to explain the Zanstra discrepancy; however, these methods give discordant results and have many uncertainties (see, for instance, Kaler 1985a, Stasińska & Tylenda 1986, and Kaler 1989).

Many authors have adopted $T_Z(\text{He II})$ as representative of the stellar temperature assuming that the Zanstra discrepancy is due to an optical depth effect (Kaler 1983b; Gleizes, Acker & Stenholm 1989; Kaler, Shaw & Kwitter 1990; Kaler & Jacoby 1991; Stanghellini, Corradi & Schwarz 1993). However, $T_Z(\text{He II})$ may not be a good indicator of the stellar temperature since it never reaches values as high as predicted by theoretical stellar evolutionary studies.

Usually the discussion of the Zanstra discrepancy in terms of an optical depth effect is based on optically thin *or* thick nebulae at 13.6 eV or 54.4 eV. It is also implicitly assumed that the nebula is optically thick at 54.4 eV at a distance to the central star smaller than that corresponding to 13.6 eV. Harman & Seaton (1966) suggest the following criteria for the complete absorption of H^0 , He^0 , and He^+ ionizing-photons: presence of [OI] lines, He

I images smaller than H I images, and He^{++} fractional abundance ≤ 0.75 , respectively. For Pottasch (1984) the Zanstra method must work for $\tau_{13.6} > 1$. Some authors define a criterion to distinguish between optically thin and thick objects using the ratio of the Zanstra temperatures, $\text{ZR} = T_Z(\text{He II})/T_Z(\text{H})$. For example, for Shaw & Kaler (1985) the nebula is optically thick to the H Lyman continuum when $\text{ZR} \lesssim 1.2$, while for $\text{ZR} \gtrsim 2.5$ and $\text{He II } \lambda 4686/\text{H}\beta \gtrsim 0.9$ it is thin for He^+ Lyman continuum photons, $T_Z(\text{He II})$ being a lower limit for the stellar temperature. The criterion used by Kaler & Jacoby (1989), based on line intensities of low-ionization lines, states that a nebula is thick when $[\text{O II}] \lambda 3727/\text{H}\beta \geq 1$ and $[\text{N II}] \lambda 6584/\text{H}\alpha \geq 1$. However, planetary nebulae can present a large range of optical depths, depending on the quantity of matter. Furthermore, even considering the central stellar radiation as a blackbody, different stellar temperatures correspond to different ratios between the number of ionizing photons with energy higher than 54.4 eV and those higher than 13.6 eV. Thus, the radial ionic distribution for H and He varies with T_* , changing the relative sizes of the H^+ and He^{++} zones with the nebula optical depth at 13.6 eV and 54.4 eV. As remarked by Stasińska & Tylenda (1986), the radiation transfer is much more complicated than assumed by the Zanstra method.

In brief, the Zanstra temperature is commonly used in the literature for planetary nebula modelizations as well as evolutionary analysis, and low optical depth must be at least a partial explanation for the Zanstra discrepancy. However, a more detailed analysis is required in order to explain the issues listed above. In this paper, a careful analysis of the effect of the nebula optical depth on the determination of the Zanstra temperatures is intended. The effects on the Zanstra temperatures due to deviations of a blackbody spectrum and due to an overabundance of He in the nebula are also discussed. The theoretical models used in our analysis are described in §2. The results for the Zanstra temperature ratio (ZR) and its behavior with the stellar temperature and with the nebula optical depth appear in §3, which also includes a comparison with values derived from PNs observations. An alternative

method to estimate the temperature of central stars of planetary nebulae is suggested and discussed in §4. The conclusions are outlined in §5.

2. Planetary nebulae models and theoretical values for the Zanstra temperatures

Models for typical planetary nebulae are generated with the photoionization code AANGABA (Gruenwald & Viegas 1992). The physical conditions of the gas are determined by solving the coupled equations of ionization and thermal balance for a spherical symmetric cloud. Several processes of ionization and recombination, as well as of gas heating and cooling, are taken into account. The transfer of the primary and diffuse radiation fields is treated in the “outward-only” approximation. For the radiation-bounded models (equivalent to completely optically thick nebulae), the calculations stop when the fractional abundance H^+/H reaches 10^{-4} , defining the maximum radius for the ionized nebula, R_{max} . Matter-bounded models (with a nebula radius less than R_{max}) will also be discussed. The input parameters are the ionizing radiation spectrum, the gas density, and the chemical abundance for the elements included in the calculations (H, He, C, N, O, Ne, Mg, Si, S, Cl, Ar, and Fe). A range of input parameters, typical of planetary nebulae (Pottasch 1984), is assumed: $T \geq 50,000K$, $L_* = 30 - 20,000 L_\odot$ and $n_H = 10^2 - 10^6 \text{ cm}^{-3}$. In order to discuss the Zanstra temperature for very hot stars, which are predicted by evolutionary models, a maximum stellar temperature of 500,000 K is adopted. A blackbody spectrum is assumed for the ionizing radiation, but the effects due to departures from this kind of spectrum will also be discussed. Concerning the chemical abundances, average values for planetary nebulae, as given by Kingsburgh & Barlow (1994), are assumed. For elements not given by these authors, the solar value is adopted (Grevesse & Anders 1989).

The He II $\lambda 4686$ and $H\beta$ fluxes obtained for the theoretical nebulae are used to derive

$T_Z(\text{H})$ and $T_Z(\text{He II})$ by the standard Zanstra method. For each set of input parameters, $T_Z(\text{H})$ and $T_Z(\text{He II})$ are calculated for different values of the nebula optical depth at the H Lyman limit.

3. Theoretical versus “observed” Zanstra temperatures

In the following section, the assumed energy distribution of the ionizing radiation is fixed (blackbody). Thus, any discrepancy between the temperature adopted for the central star, T_* , and the derived Zanstra temperatures is not due to the assumed spectrum but inherent to the method.

Our results show that $T_Z(\text{H})$ reproduces fairly well the stellar temperature for *optically thick nebulae* ionized by a star with $T_* < 150,000$ K, in agreement with Henry & Shipman (1986) and Stasińska & Tylenda (1986). For higher stellar temperatures, $T_Z(\text{H})$ is greater than T_* , and the difference between these two values increases with T_* . These results agree with those of Stasińska & Tylenda (1986). We find, however, that the deviation of $T_Z(\text{H})$ relative to T_* is slightly smaller. As already discussed by Stasińska & Tylenda (1986) the deviation of Zanstra temperatures from the stellar temperature is due to the fact that each He^{++} recombination gives more than one photon that ionizes H, and the proportion of photons ionizing He^+ increases with the stellar temperature. Furthermore, a fraction of the high energy photons are in fact absorbed by H and not by He^+ . A detailed discussion on the generation of H ionizing photons following the He^+ and He^{++} recombination can be seen in Osterbrock (1989).

3.1. Emitting volumes of H^+ , He^+ , and He^{++}

Before discussing the influence of the nebula optical depth on the derived Zanstra temperatures, it is useful to illustrate how the relative sizes of the H^+ , He^+ , He^{++} Strömgen spheres change as a function of the stellar temperature. The variation of the fractional abundances of H and He ions with the position in the nebula is shown in terms of r/R_{max} in Figure 1a (left panels) where r is the distance from the center of the nebula and R_{max} is the maximum dimension of the ionized region (see §2). The results given in Figure 1 correspond to models with $L_* = 3000 L_\odot$ and $n_H = 10^4 \text{ cm}^{-3}$. For low stellar temperatures the He^{++} Strömgen radius, $R_{He^{++}}$, is much smaller than R_{He^+} or R_{H^+} , as expected. However, as T_* increases, $R_{He^{++}}$ approaches R_{H^+} .

For a matter-bounded nebula (with a total extent less than R_{max}) the emitting zones of H^+ , He^+ , and He^{++} can be smaller than their corresponding Strömgen spheres. In this case, the emitted line intensities will be lower than those emitted by a radiation-bounded nebula. For a given reduction of the nebula extent, the size of the emitting zones of each of these ions will be differently affected, depending on the temperature of the central star. For nebulae with low stellar temperatures, a reduction of the nebula size affects mainly the H^0 , H^+ , and He^0 zones. Thus, the smaller the nebula radius, the lower $T_Z(H)$, while $T_Z(He \text{ II})$ may still be a good indicator of the stellar temperature. For increasing stellar temperatures, the volumes of the H^+ and He^{++} zones tend to be equal (Fig. 1a). In this case a reduction of the nebula size can result in a matter-bounded nebula where the H^+ and He^{++} zones are almost equally affected. In this case, both $T_Z(H)$ and $T_Z(He \text{ II})$ underestimate the stellar temperature.

3.2. The effect of the nebula optical depth

The possible underestimate of the stellar temperature, due to the fact that a nebula may not be radiation-bounded, can be discussed as an optical depth effect. If the nebula has not enough material to be radiation-bounded, its radius is smaller than R_{max} , and the nebula optical depth at a given frequency will be lower than the optical depth of a radiation-bounded nebula. The reduction of the nebula radius (creating a matter-bounded nebula) will differently affect the nebula optical depth of the H^0 , He^0 , and He^+ continua. The fractional ionic distribution of H and He ions with the optical depth at the H Lyman limit ($\tau_{13.6}$) is shown in Figure 1b. As seen in §3.1, for a radiation-bounded nebula with low T_* the He^{++} Strömgen radius is much smaller than the H^+ Strömgen radius. Thus matter-bounded nebulae can have an optical depth at the H Lyman continuum, $\tau_{13.6}$, close to unity, while the optical depth at the He^{++} Lyman limit, $\tau_{54.4}$, is much higher. The object is then optically thin to the H-ionizing photons and optically thick to the He^+ -ionizing photons, leading to ZR higher than 1. In this case, $T_Z(He\ II)$ provides a better estimate of the stellar temperature. As the stellar temperature increases, the optical depth at the H and He^+ Lyman limits tend to have similar values; both will be reduced if the radius of the nebula is smaller than that of a radiation-bounded nebula. In this case, neither $T_Z(H)$ nor T_Z is a good indicator of the stellar temperature.

The effect of the nebula optical depth on the derived Zanstra temperatures is shown in a ZR versus $T_Z(He\ II)$ plot (Figs. 2a and 2b) for the same models as in Figure 1. Each solid line corresponds to models with a given stellar temperature; the nebula optical depth decreases with increasing ZR. The curves are labeled by the stellar temperature in units of 1000 K. The dashed curves connect the results of completely optically thick models (radiation-bounded nebulae) with different stellar temperatures; these results correspond to the minimum ZR value for a given stellar temperature.

Recalling the ionic distribution shown in Figures 1a and 1b, the behavior of ZR shown by the curves in Figure 2 can be easily understood: (1) For $T_* < 150,000$ K, the He^{++} zone is inside the H^+ zone and much smaller. Matter-bounded models with decreasing $\tau_{13.6}$ would result in weaker $\text{H}\beta$ emission line, while the $\text{He II } \lambda 4686$ line is unchanged. Thus, starting at the minimum value, corresponding to the optically thick model, ZR increases while $T_Z(\text{He II})$ is practically constant. When the optical depth is low enough to affect the He^{++} zone, ZR still increases but $T_Z(\text{He II})$ decreases and the curves turn to the left; (2) For higher stellar temperatures, the decrease of $T_Z(\text{He II})$ with $\tau_{13.6}$ happens closer to the optically thick value (the volumes of the H^+ and He^{++} zones are similar) and ZR increases slowly. In each of the solid lines in Figure 2a, the points corresponding to $\tau_{13.6} = 1$, $\tau_{13.6} = 10$, and $\tau_{54.4} = 1$ are indicated, respectively, by crosses, triangles, and dots. Notice that for $T_* > 200,000$ K, a nebula can be optically thin for He^+ -ionizing photons, even for $\text{ZR} \sim 1$, though thick for photons above the H Lyman limit.

The theoretical results also show that the Zanstra method tends to underestimate the stellar temperature. The effect is larger for higher stellar temperatures, even for high optical depths. This may explain the "Zanstra wall" in the $\log L - \log T$ plot, since high-temperature stars, predicted by stellar evolutionary models, are penalized by the Zanstra method.

3.3. Confronting the theoretical results with the observations

Values for ZR and $T_Z(\text{He II})$ derived from observations for a large sample of PNs are plotted in Figure 2b in order to be compared to the theoretical results. For each object several values of the Zanstra temperatures can be found in the literature. The criteria used to select the objects and the values of the Zanstra temperatures plotted in Figure 2b are the following: (1) if the Zanstra temperatures coming from different authors are similar (difference less than 20% from the average value), their average value is taken; (2) if the

same author presents discordant data for the same object, the more recent value is taken; (3) if all the data for a given object are discordant, the object is not included. The values for both Zanstra temperatures were taken from: Martin (1981); Kaler (1983b); Pottasch (1984); Reay et al. (1984); Shaw & Kaler 1985; Viadana & de Freitas Pacheco (1985); de Freitas Pacheco, Codina & Viadana (1986); Gathier & Pottasch (1988, 1989); Gleizes et al. (1989); Jacoby & Kaler (1989); Shaw & Kaler (1989); Kaler et al. (1990); Kaler & Jacoby (1991); Méndez et al. (1992). Since we are discussing the standard Zanstra method, Zanstra temperatures corrected by the Stasińska-Tylenda effect (1986) were not included.

Most of the observational points for ZR and $T_Z(\text{He II})$ are inside the region defined by the theoretical curves that correspond to ionizing stars with a blackbody spectrum. Our results naturally explain the trend shown by the observational values: for lower H Zanstra temperatures ($\leq 100,000$ K) many planetaries may have $T_Z(\text{He II}) > T_Z(\text{H})$, i.e., $ZR > 1$, while for higher temperatures the difference between these temperatures is smaller. Such a behavior, referred to as “strange” by Pottasch (1984), induced Gathier & Pottasch (1988) to discard the optical depth explanation, since nebulae with higher stellar temperature should be older and optically thinner. The distribution of ZR versus $T_Z(\text{H})$ presents a similar trend. Notice that a decreasing ZR ratio with increasing $T_Z(\text{H})$ was obtained by Gathier & Pottasch (1988) with a sample including fewer objects.

The variation of the stellar temperatures with the optical depth in Figure 2 can solve some problems raised in the literature. One such problem is the temperature of the ionizing star of NGC 1360. The Zanstra temperatures for this object (34,900 K and 79,300 K, from the references given above) are much smaller than the temperature obtained from UV measurements (100,000 K; Pottasch et al. 1978). In Figure 2b the position of this object is in the region where the lines are crowded; the nebula is thus optically thin and the star can have a higher temperature than that given by $T_Z(\text{He II})$, as suggested by the UV data. Also,

the discussion (Kaler & Hartkopf 1981) regarding A43 (a thin and high-excitation nebula with a low He II Zanstra temperature star) and A50 (medium excitation, thick and high T_Z) must be reviewed, since, following our results (Figure 2), the central star of A43 can have a temperature much higher than $T_Z(\text{He II})$.

3.4. Other effects

In the previous section, using the results from photoionization models corresponding to a given value of the stellar luminosity and gas density and assuming a blackbody spectrum for the ionizing radiation, it was shown that the main issues concerning the Zanstra temperatures can be explained by an optical depth effect. In the following discussion the results for different values for the stellar luminosity and/or the gas density, as well as for an ionizing radiation spectrum deviating from a blackbody shape, are presented.

First, still adopting a blackbody spectrum, we discuss the results corresponding to the whole range of adopted values for the stellar luminosity and gas density. We verified that, as long as $T_* < 200,000$ K, the behavior of ZR with $T_Z(\text{He II})$ is the same as discussed in §§3.1 and 3.2. For $T_* \geq 200,000$ K and $\tau_{13.6} > 1$, models with low stellar luminosities ($\leq 100 L_\odot$) *and* low gas density ($< 10^3 \text{ cm}^{-3}$) can give Zanstra temperatures lower than those obtained with the standard models discussed above. The differences between the Zanstra and stellar temperatures increase with increasing T_* and decreasing values for L_* and n_H . For example, for $T_* = 300,000$ K, a maximum difference occurs for $\tau_{13.6} \sim 10$, $L_* = 10 L_\odot$, and $n_H = 100 \text{ cm}^{-3}$, when both Zanstra temperatures decrease by $\sim 20 \%$, increasing the difference between Zanstra and effective stellar temperatures. However, only a few PNs would have such low stellar luminosities *and* gas densities.

A number of authors explain the Zanstra discrepancy by an excess of photons with

energy above 54.4 eV in the ionizing spectrum. This could explain the high values of $T_Z(\text{He II})$ compared to $T_Z(\text{H})$. As discussed by Henry & Shipman (1986), observations and models imply an excess of photons beyond the He^+ threshold in numerous planetary nebula nuclei. Such an excess could be produced by a stellar atmosphere with subsolar He abundances and would lead to $T_Z(\text{He II})$ higher than $T_Z(\text{H})$ when compared with models where a blackbody is assumed.

To show the effect of a spectrum presenting an excess of high-energy photons above 54.4 eV, we discuss the results of photoionization models with an ionizing radiation spectrum of a pure H atmosphere (Wesemael et al. 1980). For example, for a completely optically thick nebula around a 150,000 K star, $T_Z(\text{He II})$ is 6% higher and $T_Z(\text{H})$ is 13% lower compared to the corresponding blackbody results. For decreasing optical depths, the curves tend rapidly to those corresponding to blackbody models. In brief, only for completely optically thick nebulae an excess of high energy photons will affect (in a small amount) the calculated Zanstra temperatures.

3.5. Outsiders

Some objects shown in Figure 2b are outside the area covered by the models. These nebulae can be either above the area limited by the curves or below it. Those above the curves limiting the high values of ZR could be explained by an error in $T_Z(\text{He II})$ of the order of 5 - 10 %. However, checking more carefully, it can be verified that most of these nebulae are Abell nebulae, including NGC 246, the prototype of the class (Abell 1966). Observations of the central star of some of these nebulae show characteristics of high stellar temperatures; furthermore, the nebulae have low surface brightness and large angular diameter (Abell 1966). These objects are probably in an advanced evolutionary stage. Some of them are known to have very high nebular He abundance in their inner regions (Jacoby & Ford 1983;

Guerrero & Manchado 1996). Calculations for He-rich nebulae show that abundances up to $\text{He}/\text{H} = 0.25$ can explain the positions of the nebulae lying above the limiting curves in Figure 2b. Results for $T_* = 150,000$ K and $\text{He}/\text{H} = 0.20$ are shown in Figure 3 by the dot-dashed line. Note that for high optical depths the curves for the same stellar temperature but different He abundance are superposed. For $T_* = 150,000$ K, the curves separate from each other when $\tau_{13.6} < 2.5$. The reason is that increasing the He abundance, the He^{++} zone decreases relative to the H^+ zone by 36% in volume for this value of T_* . Thus, a decrease of the nebula optical depth will only affect the He^{++} zone [and consequently $T_Z(\text{He II})$] when $T_Z(\text{H})$ is very low. Thus, the area covered by the models stretches toward higher ZR, including the high ZR objects.

Large nebulae have been studied in detail by Kaler and collaborators (Kaler 1981, 1983b; Kaler & Feibelman 1985; Kaler et al. 1990). For many of these nebulae the color temperature obtained from UV observations are well above their Zanstra temperatures. This is consistent with our results (Fig. 2b) that show that the stellar temperature can be higher than $T_Z(\text{He II})$. Because of their large diameters, the density in the Abell or other large nebulae may be smaller than the one assumed for the standard models. However, as mentioned above, results for the Zanstra temperatures with different densities are similar.

The above results do not necessarily mean that all the nebulae lying above the limiting curves are He-rich, or, inversely, that all He-rich nebulae have positions above the curves plotted in Figure 2. The same questions can be asked regarding the nebulae size. From the 19 nebulae above the curves, 14 have calculated or limiting values for the abundance; from these, only one have He abundance definitively below the average value for planetary nebulae as given by Kingsburgh & Barlow (1994). However, He-rich nebulae are also found in regions of higher optical depth in the diagrams. Regarding the size, all nebulae above the curves, except two (Hu 1-2, with 0.018 pc, and Cn 1-2, with no value calculated for

the radius) have radius larger than 0.15 pc (Cahn, Kaler, & Stanghellini 1992). But large nebulae are spread everywhere in the diagrams. In brief, large and/or He-rich nebulae can be found in any location on the diagram, but most of those above the limiting curves are large and He-rich.

As discussed below, a He-rich atmosphere may provide the explanation for the outsiders with very low ZR. A high He abundance in the inner parts of a nebula can indicate a high He in the upper layers of the star, including their atmospheres. The energy spectrum of a He-rich atmosphere will show a deficit of high energy ($E > 54.4$ eV) photons because of the contribution of He to the stellar continuum opacity. Henry & Shipman (1986) discarded a high He abundance in the stellar atmosphere, since they do not explain the Zanstra discrepancy shown by most PNs ($ZR > 1$), concluding that the atmospheres have subsolar He abundance (and an excess of high-energy photons). Our results show that the main effect originating the Zanstra discrepancy is the optical effect. However, photoionization models assuming an ionizing spectrum with a deficit of photons with energy higher than 54.4 eV indicate that ZR is less than unity in the optically thick case, lowering the limit defined by the models. Results for models with various stellar temperatures and an ionizing spectrum showing a deficit of a factor of five in the flux of high-energy photons, relative to a blackbody, are shown by the dotted lines in Figure 3. The corresponding blackbody results are shown by the solid lines. Thus, such ionizing spectrum, presenting a deficit of high-energy photons, may explain the outsiders with low ZR.

4. Stellar temperatures and the He II/He I line intensity ratio

The results presented in Figures 2 and 3 can be used to obtain the nebula optical depth as well as a value for the stellar temperature more accurately than that given by the Zanstra temperatures. However, for optically thin nebulae, the curves are crowded up and

the temperature is not well defined. The same occurs for high stellar temperatures, even at high optical depths. Thus, another method for obtaining the stellar temperature is required.

Since the optical depth is a major factor of the stellar temperature determination, line intensity ratios produced by ions in different ionization stages can be used to distinguish nebulae with different optical depths. When plotted against $T_Z(\text{He II})$, many of these line ratios also show a crowding of the curves corresponding to different models. The best ratio discriminating the results for different models is the ratio between He II and He I line intensities. The results for He II $\lambda 4686/\text{He I } \lambda 5786$ and He II $\lambda 4686/\text{He I } \lambda 4471$ versus $T_Z(\text{He II})$ are presented in Figures 4a and 4b, respectively, for the same models of Figures 1 and 2. Notice that, for a given range of T_* and $\tau_{13.6}$, particularly for low optical depths, the curves in Figure 4 are more widely spaced and provide a better determination of these parameters than the curves shown in Figure 2.

Regarding the nebulae in the crowded region of Figure. 2b, for which there is an uncertainty in the determination of T_* , the stellar temperature may be obtained from Figure 4. Besides the 19 nebulae above the curves in Figure 2b, 28 nebulae are in the region where the results for T_* are just lower limits. For all these nebulae, only 20 have measured intensities for He II and He I lines. With intensities taken from the literature (Torres-Peimbert & Peimbert 1977; Aller & Czyzak 1979, 1983; Jacoby & Ford 1983; Kaler 1983a; Kaler 1985b; Manchado, Mampaso & Pottasch 1987; Peimbert & Torres-Peimbert 1987; Kaler et al. 1990; Acker et al. 1991; de Freitas Pacheco, Maciel & Costa 1992; Stanghellini, Kaler & Shaw 1994; Kingsburgh & Barlow 1994), the stellar temperatures obtained from Figure 4 are higher than $T_Z(\text{He II})$ by about 10% in general but can reach 32%. For the selected nebulae, the maximum observed value for He II $\lambda 4686/\text{He I } \lambda 5786$ is 1.85dex for NGC 4361 (there is also a measured lower limit for its central region of 2.25dex). For He II $\lambda 4686/\text{He I } \lambda 4471$ the maximum measured ratio is 2.37dex, for NGC 2022. Higher ratios, corresponding to lower

optical depths, are not measured since He I is too faint to be detected. Notice that for high T_* ($> 150,000$ K), He I can be very faint even for $\tau_{13.6}$ higher than unity, since, as shown in Figure 1, the higher T_* the smaller the He^+ region. So, for nebulae with low ZR and no detected He I line, the stellar temperature can be much higher than $T_Z(\text{He II})$. As long as the He II Zanstra temperature is known, this alternative method can be used and may be considered as a second-order approximation to the stellar temperature, providing values closer to the real stellar temperature, even for nebulae showing a high He II/He I line ratio.

5. Conclusions

One of the problems concerning the understanding of PNs and their evolution is the determination of their stellar temperature. The Zanstra method is generally used, although the H and He II Zanstra temperatures may be discrepant and may underestimate the stellar temperature. Many authors suggested that the dominant mechanism explaining the Zanstra discrepancy is the optical depth. Because $T_Z(\text{He II})$ is less affected by the nebula optical depth, these authors suggest adopting $T_Z(\text{He II})$ as the true stellar temperature. Besides the importance of a good determination of the stellar temperature of planetary nebulae for the analysis of the emission-line spectrum and the chemical abundance determination of PNs, let us recall that the position of the central star of PN on the H-R diagram is a crucial test of evolutionary models from the AGB to the white dwarf stages.

Here the nebula optical depth effect is analyzed in detail using photoionization models. From the theoretical $\text{H}\beta$ and He II $\lambda 4686$ line intensities, the Zanstra temperatures are calculated and compared to the adopted stellar temperatures for a variety of models with different optical depths. Our results show that the nebula optical depth is the main factor explaining the behavior of the Zanstra temperatures with the stellar temperature. The Zanstra discrepancy mainly occurring for low stellar temperatures is clearly explained by the

changes induced by the optical depth in the relative ionic distribution in the nebula (Figs. 1 and 2). Another consequence is that even the He II Zanstra temperature underestimates T_* , mainly for nebulae with high-temperature stars. The results showing that the stellar temperature can be higher than $T_Z(\text{He II})$ are consistent with UV data for the central stars of PNs (Pottasch et al. 1978; Kaler & Feibelman 1985). The relation between the Zanstra temperature ratio ZR and $T_Z(\text{He II})$ (Fig. 2) can be used to obtain a more accurate estimate of the stellar temperature. However, for nebulae with very high stellar temperature and/or small optical depths the theoretical results for different stellar temperatures and optical depths are crowded and the method is uncertain. For these nebulae, a better determination of these parameters can be obtained from a plot of He II/He I versus $T_Z(\text{He II})$ (Figs. 4).

An important source of uncertainty is related to the kind of observations used to determine the stellar temperature. Observed line intensities do not always refer to the whole nebula since line ratios are usually obtained from observations with a narrow slit crossing the nebula. Furthermore, the nebula may be inhomogeneous and the optical depth anisotropic. That is, the nebula can be optically thick in some direction but optically thin in others. This is a very important point not addressed in this paper. Line intensity ratios obtained with a narrow slit, or in a given position of a nebula, may not correspond to the ratio for the entire nebula (Gruenwald, Viegas, & Broguière 1997; Gruenwald & Viegas 1998).

Cases of very high or very low ZR can be explained by the coupled effect of optical depth and over- or under-abundance of He in the stellar atmosphere, which affects the ionizing spectrum. A nebular overabundance of He relative to solar values and a low optical thickness explain the very high discrepancy shown by some Abell planetaries. On the other hand, ZRs less than unity are characteristic of optically thick PNs with undersolar He abundance.

Stanghellini et al. (1993) state that since bipolar nebulae have $T_Z(\text{He II}) \sim T_Z(\text{H})$, they are thicker than other nebulae. However, central stars of bipolar nebulae are known to have

high temperatures (Corradi & Schwarz 1995). From our results (Figure 2) these nebulae are expected to have $T_Z(\text{He II}) \sim T_Z(\text{H})$, even if they are not completely optically thick.

Finally, the “Zanstra wall” in the $\log L - \log T$ diagram (Shaw and Kaler 1989) is related to the fact that the Zanstra method underestimates the stellar temperature and this effect is larger for high-temperature stars. Consequently, the lack of high-temperature stars (predicted by the evolutionary models) in the $\log L - \log T$ diagram can be easily understood.

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Fig. 1.— Fractional abundance distribution of H and He ions: dependence with (a) the radial distance from the center, in units of the maximum radius and (b) the optical depth in 13.6 eV. The figures are labeled by the stellar temperature adopted in the models. Thick and thin solid lines correspond, respectively, to the fractional abundances of H^0 and H^+ relative to H, while the fractional abundances of He^0 , He^+ , and He^{++} relative to He are given, respectively, by the dotted, dashed, and dot-dashed lines.

Fig. 2.— Ratio of Zanstra temperatures vs. $T_Z(\text{He II})$ for the same models as in Fig. 1. Each solid line is labeled by the corresponding stellar temperature in units of 1000 K. For each solid line, increasing optical depths correspond to decreasing ZR. The dashed curves connect the results of completely optically thick models with different stellar temperatures. (a) Results characterized by $\tau_{13.6} = 1$ or $\tau_{13.6} = 10$ or $\tau_{54.4} = 1$ are indicated, respectively, by crosses, triangles, and dots; (b) dots represent the Zanstra temperatures derived from observations.

Fig. 3.— Effects of the nebula He abundance and of a departure from a blackbody ionization spectrum on the Zanstra temperatures. Solid lines correspond to the same models as in Fig. 1 ($\text{He}/\text{H} = 0.115$). Results for a model with $T_* = 150,000$ K and a higher abundance ($\text{He}/\text{H} = 0.200$) are indicated by the dot-dashed line. Dotted lines show the results for models with a deficit of high-energy photons in the ionizing spectrum for various T_* . The dashed lines join the completely optically thick models.

Fig. 4.— He II/He I line intensity ratios versus $T_Z(\text{He II})$. The solid lines are labeled by the stellar temperature in units of 1000 K. Models for $\tau_{13.6} = 1$ or $\tau_{13.6} = 10$ or $\tau_{54.4} = 1$ are indicated, respectively, by the dotted, dot-dashed, and dashed lines.